Exhibit C

Aboelmagd Noureldin
Tashfeen B. Karamat · Jacques Georgy

Fundamentals of Inertial Navigation, Satellite-based Positioning and their Integration

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Department of Electrical and Computer
Engineering
Royal Military College of Canada/
Queen's University
Kingston
Canada

Trusted Positioning Inc.
Calgary
Canada

Tashfeen B. Karamat
Department of Electrical
and Computer Engineering
Queen's University
Kingston
Canada

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Figure 7.2 shows the discrete-time system corresponding to Eqs. (7.1) and (7.2).

The state transition matrix (STM) Φ represents the known dynamic behavior of the system (in this case the INS error model) which relates the state vector from epoch $k-1$ to k. Given the dynamic coefficient matrix F of a continuous time system the STM is

$$\Phi = \exp(F\Delta t) \tag{7.3}$$

To linearize this for use by KF we take the first two terms of the Taylor series expansion of the equation as follows

$$\Phi = (I + F\Delta t) \tag{7.4}$$

where I is identity matrix and Δt is sampling interval.

7.1.1 KF Assumptions

Kalman filtering relies on the following assumptions (Maybeck 1979; Minkler and Minkler 1993).

1. The system (both the process and the measurements) can be described by linear models.
2. The system noise $\mathbf{w}_k$ and the measurement noise $\boldsymbol{\eta}_k$ are uncorrelated zero-mean white noise processes with known auto covariance functions, hence

$$E[\mathbf{w}_k] = 0, \quad E[\boldsymbol{\eta}_k] = 0 \quad \forall k \tag{7.5}$$

$$E[\mathbf{w}_k \boldsymbol{\eta}_j^T] = 0 \quad \forall k, j \tag{7.6}$$

$$E[\mathbf{w}_k \mathbf{w}_j^T] = \begin{cases} Q_k, & k = j \\ 0 & k \neq j \end{cases} \tag{7.7}$$

$$E[\boldsymbol{\eta}_k \boldsymbol{\eta}_j^T] = \begin{cases} R_k, & k = j \\ 0 & k \neq j \end{cases} \tag{7.8}$$

   where Q_k and R_k are known positive definitive matrices. In INS/GPS integration, Q_k represents the covariance matrix of the system noise associated with the INS errors, and R_k represents the covariance matrix of the measurement noise associated with the GPS position and velocity updates.
3. The initial system state vector $\mathbf{x}_0$ is a random vector uncorrelated to both the process and measurement noises, hence

$$E[\mathbf{x}_0 \mathbf{w}_k^T] = 0, \quad E[\mathbf{x}_0\, \boldsymbol{\eta}_k^T] = 0 \quad \forall k \tag{7.9}$$